

# Time Series Modelling with Unobserved Components: Errata

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## Chapter 1

**Page 4, first equation in display** The loss function should be

$$\ell(E) = \begin{cases} 19E & \text{for } E \geq 0, \\ -2E & \text{for } E < 0. \end{cases}$$

**Page 4, line 6 from the bottom** The line should read ‘better reasons suggest the use of different loss curves.’

**Page 5, line 10 of the *Proof*** Substitute ‘by Property 2.’ with ‘by Property 5.’.

**Page 6, line 11 of the *Proof*** Correct the last sentence in this line as ‘Therefore, the expectations of  $(Y|X)$  and  $Y$  are equal.’

**Page 6, last line of the *Proof*** After the very last equal sign, it should read  $\mathbb{E}(\text{Var}[Y|X])$  instead of just  $\text{Var}[Y|X]$ .

**Page 8, line 5** Substitute ‘ $n + 1$ ’ with ‘ $m + 1$ ’.

**Page 9, point 1 in Theorem 1.4** The zero should be bold:  $\mathbf{0}$ , because it is a vector (of zeros), not a scalar.

**Page 9, point 4 in Theorem 1.4** The  $a$  and  $b$  should be matrices and  $c$  should be a vector. The equation should read:

$$\mathbb{P}[\mathbf{A}\mathbf{Y} + \mathbf{B}\mathbf{Z} + \mathbf{c}|\mathbf{X}] = \mathbf{A}\mathbb{P}[\mathbf{Y}|\mathbf{X}] + \mathbf{B}\mathbb{P}[\mathbf{Z}|\mathbf{X}] + \mathbf{c}$$

**Page 9, point 7 in Theorem 1.4** The final part of the second line of the equation should read  $(\mathbf{Z} - \mathbb{P}[\mathbf{Z}|\mathbf{X}])$  ( $P$  should be  $\mathbb{P}$ ).

**Page 10, line 16** The second line of the equation after *Projection on orthogonal variables* should read:

$$= \boldsymbol{\mu}_Y + \boldsymbol{\Sigma}_{YX}\boldsymbol{\Sigma}_{XX}^{-1}(\mathbf{X} - \boldsymbol{\mu}_X) + \boldsymbol{\Sigma}_{YZ}\boldsymbol{\Sigma}_{ZZ}^{-1}(\mathbf{Z} - \boldsymbol{\mu}_Y)$$

**Page 11, first line** In the middle of the line substitute with ‘ $\mathbb{P}[\mathbf{Z}|\mathbf{X}, \mathbf{Z}] = \mathbf{Z}$ ’.

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## Chapter 2

**Page 19, Theorem 2.2** The property name ‘(Positivity of variance)’ should be substituted with ‘(Nonnegativity of variance)’.

**Page 19, lines 11 and 9 from the bottom and last display** In all cases the mathematical symbols ‘ $\mathbf{y}$ ’ and ‘ $\mathbf{y}_m$ ’ should be substituted with the capitalised ‘ $\mathbf{Y}$ ’.

**Page 27, 2nd line of 2nd display** The linear prediction of  $V_t$  should read

$$\mathbb{P}[V_t|V_{t-1}, V_{t-2}] = \cos(\lambda)V_{t-1} + \cos(2\lambda)V_{t-2},$$

as correctly reported in the following display.

**Page 29, 1st line of Section 2.3** Substitute ‘demographics’ with ‘demographic’.

**Page 36, 1st line** After ‘From Wold’s theorem’ add a reference to the theorem number like ‘(Theorem 2.7)’.

**Page 38, Definition 2.14** Substitute the beginning of the definition with ‘The ARMA( $p, q$ ) process (2.7) is *invertible*’.

**Page 44, 2nd line of 3rd display** The line should read

$$\Theta_Q(\mathbb{B}) = 1 + \Theta_1\mathbb{B}^s + \Theta_2\mathbb{B}^{2s} + \dots + \Theta_Q\mathbb{B}^{Qs}.$$

**Page 44, 2nd line before Section 2.5** the second word should be ‘model’ and not ‘mode’.

**Page 44, last line in display of Definition 2.15** The footer of ‘ $\mathbf{Y}_{t-h}$ ’ should not be boldface: ‘ $\mathbf{Y}_{t-h}$ ’.

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## Chapter 3

**Page 57, 7th line from the bottom** The second equation in the display should read

$$\mu(x_{t+1}) = \mu(x_t) + \lambda_S^{-\frac{1}{2}} \sigma \int_{x_t}^{x_{t+1}} W(s) ds,$$

**Page 58, first equation** It should read

$$\begin{bmatrix} \eta_t \\ \zeta_t \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \frac{\sigma^2}{\lambda_S} \begin{bmatrix} \frac{1}{3}\delta_t^3 & \frac{1}{2}\delta_t^2 \\ \frac{1}{2}\delta_t^2 & \delta_t \end{bmatrix} \right).$$

**Page 59, Figure 3.5** In both plots, the numbers below the  $x$ -axis should read  $(0, 1/4, 1/2, 3/4, 1)$  and not  $0, \pi/4, \pi/2, 3\pi/4, \pi$ .

**Page 61, 4th line from the bottom** The equation in Theorem 3.1 should read

$$\mathbb{E} \left[ \boldsymbol{\psi}_{t+h} \boldsymbol{\psi}_t^\top \right] = \frac{\rho^{|h|} \sigma_\kappa^2}{1 - \rho^2} \mathbf{R}(h\lambda),$$

**Page 61, last two lines** The sentence should read: ‘The optimal predictor of the stochastic cycle at time  $t + h$  (with  $h > 0$ ) [...]’

**Page 63, 10th line from the bottom** The equation display after ‘For the autocovariance function’ should read

$$\begin{aligned} \mathbb{E} \left[ \boldsymbol{\psi}_{t+h} \boldsymbol{\psi}_t^\top \right] &= [\text{as in book}] \\ &= \mathbf{R}(h\lambda) \rho^h \boldsymbol{\Sigma}_\psi \\ &= \frac{\rho^h \sigma_\kappa^2}{1 - \rho^2} \mathbf{R}(h\lambda). \end{aligned}$$

**Page 68, Lemma 3.2** The beginning should read ‘Let  $f_t : \mathbb{Z} \mapsto \mathbb{R}$  be a function of period  $s \in \mathbb{Z}$  (i.e.,  $f_{t+s} = f_t$  for all  $t$ ).’

**Page 81, central equation, page 82, first equation** In the first line of those equations the summation starts from  $j = 1$  and not from  $j = 2$ .

## Chapter 5

**Page 92, 4th line** The first item in the list should read

- $\mathbb{E}[\boldsymbol{\alpha}_1] = \mathbf{a}_{1|0}$  and  $\mathbb{E}[(\boldsymbol{\alpha}_1 - \mathbf{a}_{1|0})(\boldsymbol{\alpha}_1 - \mathbf{a}_{1|0})^\top] = \mathbf{P}_{1|0}$ ;

**Page 93, 6th line in Example 5.1** There should be a comma after  $\phi$ : ‘ $\mathbf{T}_t = \phi, \mathbf{H}_t = 0$ ’, furthermore ‘ $\mathbf{Q}_t = \sigma^2$ ’.

**Page 96, 3rd line of Example 5.4** ‘though’ should be ‘through’.

**Page 97, 4th line from the bottom** After the sentence ‘The same process can also be written as’ the matrix equation has wrong indexes in the last vector after the ‘+’ symbol. The equation should read

$$\underbrace{\begin{bmatrix} \alpha_{1,t+1} \\ \alpha_{2,t+1} \\ \vdots \\ \alpha_{r-1,t+1} \\ \alpha_{r,t+1} \end{bmatrix}}_{\boldsymbol{\alpha}_{t+1}} = \underbrace{\begin{bmatrix} \phi_1 & 1 & \dots & 0 & 0 \\ \phi_2 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \dots & \ddots & \vdots \\ \phi_{r-1} & 0 & \dots & 0 & 1 \\ \phi_r & 0 & \dots & 0 & 0 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} \alpha_{1,t} \\ \alpha_{2,t} \\ \vdots \\ \alpha_{r-1,t} \\ \alpha_{r,t} \end{bmatrix}}_{\boldsymbol{\alpha}_t} + \underbrace{\begin{bmatrix} 1 \\ \theta_1 \\ \vdots \\ \theta_{r-2} \\ \theta_{r-1} \end{bmatrix}}_{\boldsymbol{\nu}_t} \boldsymbol{\nu}_t$$

**Page 98, last display in the page** the transition matrix of the ARI(1,1) model should read

$$\begin{bmatrix} (1 + \phi_1) & 1 \\ -\phi_1 & 0 \end{bmatrix}$$

**Page 99, display in Example 5.6** The transition matrix of the ARIMA(0,1,1)(0,1,1) model should read

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**Page 101, line 11** ‘can be dropped’ should read ‘is to be dropped’.

**Page 108, the two matrix equations** In the first and second matrix equations, there is an  $r$  instead of  $v$  in the vectors on the lhs of the equal sign. In the second matrix equation, in the vector on the rhs of the ‘+’ sign the indexing is wrong. The two equations should read respectively as

$$\underbrace{\begin{bmatrix} \alpha_{1,t+1} \\ \alpha_{2,t+1} \\ \vdots \\ \alpha_{v-1,t+1} \\ \alpha_{v,t+1} \end{bmatrix}}_{\alpha_{t+1}} = \underbrace{\begin{bmatrix} \delta_1 & \delta_2 & \dots & \delta_{v-1} & \delta_v \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \ddots & \dots & \vdots & \vdots \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} \alpha_{1,t} \\ \alpha_{2,t} \\ \vdots \\ \alpha_{v-1,t} \\ \alpha_{v,t} \end{bmatrix}}_{\alpha_t} + \underbrace{\begin{bmatrix} X_t \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{d_t}$$

and

$$\underbrace{\begin{bmatrix} \alpha_{1,t+1} \\ \alpha_{2,t+1} \\ \vdots \\ \alpha_{v-1,t+1} \\ \alpha_{v,t+1} \end{bmatrix}}_{\alpha_{t+1}} = \underbrace{\begin{bmatrix} \delta_1 & 1 & \dots & 0 & 0 \\ \delta_2 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \dots & \ddots & \vdots \\ \delta_{v-1} & 0 & \dots & 0 & 1 \\ \delta_v & 0 & \dots & 0 & 0 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} \alpha_{1,t} \\ \alpha_{2,t} \\ \vdots \\ \alpha_{v-1,t} \\ \alpha_{v,t} \end{bmatrix}}_{\alpha_t} + \underbrace{\begin{bmatrix} \omega_0 X_t \\ \omega_1 X_t \\ \vdots \\ \omega_{v-2} X_t \\ \omega_{v-1} X_t \end{bmatrix}}_{d_t}$$

**Page 112, Equation 5.9** The first line of the equation should read

$$\hat{\mathbf{Y}}_{t|t-1} = \mathbb{P}[\mathbf{Y}_t | \mathcal{Y}_{t-1}] = \mathbf{Z}_t \mathbf{a}_{t|t-1} + \mathbf{c}_t$$

**Page 113, lines 5-10 (first sequence of equation of the proof)** Every symbol  $d_t$  should be substituted with the symbol  $c_t$ .

**Page 113, line 7 from the bottom** ‘to can compute’ should read ‘to compute’.

**Page 113, last display** The first expectation after the equal sign should read

$$\mathbb{E}[\boldsymbol{\nu}_{t-1}\boldsymbol{\nu}_{t-1}^\top]$$

**Page 116, equation 5.12** In the first line of this equation there should be a subscript  $t$  just after the matrix  $\mathbf{K}$ :

$$\mathbf{a}_{t+1|t} = \mathbf{T}_t\mathbf{a}_{t|t-1} + \mathbf{d}_t + \mathbf{K}_t\boldsymbol{\nu}_t,$$

**Page 118, line 3 from the bottom** ‘Consistent with’ should read ‘Consistently with’.

**Page 120, first line of first display** The line should begin with

$$\mathbf{a}_{n|n} = \mathbf{a}_{n|n-1} +$$

**Page 121, line 2 from the bottom** The symbol ‘ $P_{t|n}$ ’ should be substituted with ‘ $a_{t|n}$ ’.

**Page 126, line 1 after first display** There should be a blank space between ‘equation’ and ‘(5.18)’.

**Page 126, Equation 5.20** There should be no transposition after any matrix  $\mathbf{T}$ . Thus the final formula should read:

$$\text{vec}(\mathbf{P}^*) = (\mathbf{I} - \mathbf{T}^* \otimes \mathbf{T}^*)^{-1} \text{vec}(\mathbf{Q}^*).$$

**Page 126, last line** The formula ‘ $\tau = \infty,$ ’ at the end of the line should read ‘ $\tau = \infty.$ ’.

**Page 128, Gaussian density in the middle of the page** Inside the exponential, the transpose sign ( $^\top$ ), should be on the first instance of  $(\mathbf{y}_t - \hat{\mathbf{y}}_{t|t-1})$  and not on the second.

**Page 133, line 11 from the bottom** The line should read ‘and (5.25) reduces to (5.22)’.

**Page 134, Table 5.1** The second column of the title row should read  $(0, \infty)$ , that is the interval should be open on both sides. The element on the last row of the last column (Jacobian of the inverse logit transform) should read

$$b \exp(-\psi) [1 + \exp(-\psi)]^{-2}$$

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## Chapter 6

**Page 138-139** Here, first the dot over the symbol is used to denote the variables *before* the log-transform, but after Figure 6.1 the dot over the symbol is used to denote the variables *after* the log-transform. Invert the use of the dot before or after the figure.

**Page 138, Figure 6.1** The last word in the caption should be ‘left’.

**Page 139, first line** ‘added’ should read ‘addend’.

**Page 140, line 20** ‘lower’ should be ‘slower’.

**Page 140, line 6 from the bottom** ‘standard deviation’ should read ‘standard deviations’.

**Page 140, line 5 from the bottom** ‘for grid  $\lambda$ -values’ should read ‘for a grid of  $\lambda$ -values’.

**Page 142, Figure 6.3** In the caption the correct values for  $\lambda$  are ‘( $\lambda = -0.25, 0.00, 1.00$ )’.

**Page 143, lines 9-10 from the bottom** ‘while it gives’ should read ‘while they give’.

**Page 148, line 5 from the bottom** ‘exact algorithm’ should read ‘exact algorithms’.

**Page 154, footnote 3** ‘in finite sample’ should read ‘in finite samples’.

**Page 156, 2nd line in cycle definition (mid-page)** ‘for orthogonal’ should read ‘for the orthogonal’.

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## Chapter 7

**Page 165, line 2 from the bottom and page 166, line 17** ‘ $r$ ’ should read ‘ $K_\mu$ ’.

**Page 166, line 5 from the bottom** ‘ $N \times 1$ ’ should read ‘ $K_\beta \times 1$ ’.

**Page 168, lines 2-3** ‘ $N_\mu$ ’ should read ‘ $K_\mu$ ’.

**Page 172, first line** ‘ $K_\beta$ ’ should read ‘ $K_\psi$ ’.

**Page 173, first line** The vector should read ‘[0.968 0.032 0.000 0.000]’

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## Chapter 8

**Page 179, line 10 from the bottom** ‘treatment’ should read ‘treatments’.

**Page 180, third display** the condition ‘ $0 < j < \frac{1}{2}$ ’ is wrong and it should be substituted with ‘ $0 < j < \frac{n}{2}$ ’.

**Page 183, line 18** ‘which no not show’ should read ‘which does not show’.

**Page 190, caption of figure 8.4** In the 2nd last line ‘(2, 4) (long dashes)’ should read ‘(2, 3) (long dashes)’.

**Page 193, line 3 from the bottom** delete one ‘to’ in ‘equal to to (2,1)’.

**Page 195, line 2 from the bottom** add the word ‘orthogonal’ after ‘reciprocally’.

## Chapter 9

**Page 207, last formula** The matrix should read

$\mathbf{P}_{1|0} =$

$$\begin{array}{l}
 \mu_1 \\
 \mu_2 \\
 \beta_1 \\
 \beta_2 \\
 \psi_1 \\
 \psi_2 \\
 \psi_1^* \\
 \psi_2^* \\
 \varepsilon_1 \\
 \varepsilon_2 \\
 \mu_2^{(-1)} \\
 \mu_2^{(-2)} \\
 \psi_2^{(-1)} \\
 \psi_2^{(-2)} \\
 \varepsilon_2^{(-1)} \\
 \varepsilon_2^{(-2)}
 \end{array}
 \begin{bmatrix}
 \mu_1 & \mu_2 & \beta_1 & \beta_2 & \psi_1 & \psi_2 & \psi_1^* & \psi_2^* & \varepsilon_1 & \varepsilon_2 & \overset{(-1)}{\mu_2} & \overset{(-2)}{\mu_2} & \overset{(-1)}{\psi_2} & \overset{(-2)}{\psi_2} & \overset{(-1)}{\varepsilon_2} & \overset{(-2)}{\varepsilon_2} \\
 \infty & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \infty & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & \infty & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & \infty & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & \sigma_{11}^\psi & \sigma_{12}^\psi & 0 & 0 & 0 & 0 & 0 & 0 & \rho_1 \sigma_{12}^\psi & \rho_2 \sigma_{12}^\psi & 0 & 0 \\
 0 & 0 & 0 & 0 & \sigma_{21}^\psi & \sigma_{22}^\psi & 0 & 0 & 0 & 0 & 0 & 0 & \rho_1 \sigma_{22}^\psi & \rho_2 \sigma_{22}^\psi & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{11}^\psi & \sigma_{12}^\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{21}^\psi & \sigma_{22}^\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{11}^\varepsilon & \sigma_{12}^\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{21}^\varepsilon & \sigma_{22}^\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \infty & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \infty & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & \rho_1 \sigma_{21}^\psi & \rho_1 \sigma_{22}^\psi & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{22}^\psi & \rho_1 \sigma_{22}^\psi & 0 & 0 \\
 0 & 0 & 0 & 0 & \rho_2 \sigma_{21}^\psi & \rho_2 \sigma_{22}^\psi & 0 & 0 & 0 & 0 & 0 & 0 & \rho_1 \sigma_{22}^\psi & \sigma_{22}^\psi & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{22}^\varepsilon & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{22}^\varepsilon
 \end{bmatrix}$$

**Page 208, lines 1-2** Substitute with ‘Furthermore,  $\rho_h$  represents the autocorrelation function of  $\{\psi_t\}$ , which is given by  $\rho_h = \rho^h \cos(h\lambda)$ , as seen in Section 3.3.’

**Page 208, line 6 from the bottom** ‘GD ’ should read ‘GDP’.

**Page 214, second display** In the matrix  $\mathbf{Q}$  the second last element on the main diagonal should be ‘ $\sigma_\zeta^2 \mathbf{I}_{32}$ ’ and not just ‘ $\mathbf{I}_{32}$ ’.

## Chapter 10

**Page 220, line 13** The second part of the line should read ‘The trend order  $d \in \{1, 2, 3, 4\}$  is used to build a’.

**Page 223, line 9** There should be a carriage return before ‘std.error’

**Page 241, line 6** ‘modes’ should read ‘models’.

**Page 241, line 9** ‘most popular softwares’ should read ‘most popular packages’.

**Page 241, line 4 from the bottom** ‘treatment of the initial conditions’ should read ‘treatment of diffuse initial conditions’.

**Page 243, line 13 from the bottom** correct with ‘parameters in the vector `pars` into the model `model` and returns the updated model.’.

**Page 243, line 11 from the bottom** correct with ‘values marked as NA on the main diagonals of the time-invariant covariance matrices `Q` and `H`.’.

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## Bibliography

**Page 247, 2nd item** correct ‘euro area’ with ‘Euro area’.

**Page 248, 3rd item** remove curly brackets from ‘KPSS’.